Section 5.4 – Sum and Difference Formulas

Don't worry, you do not have to memorize the following formulas,
but you have to know how to use them

$$\sin(\theta+\beta)=\sin\theta\cos\beta+\cos\theta\sin\beta$$

$$\sin(\theta-\beta)=\sin\theta\cos\beta-\cos\theta\sin\beta$$

$$\cos(\theta+\beta)=\cos\theta\cos\beta-\sin\theta\sin\beta$$

$$\cos(\theta-\beta)=\cos\theta\cos\beta+\sin\theta\sin\beta$$

$$\tan(\theta+\beta)=\frac{\tan\theta+\tan\beta}{1-\tan\theta\tan\beta}$$

$$\tan(\theta-\beta)=\frac{\tan\theta-\tan\beta}{1+\tan\theta\tan\beta}$$

Ex. 1) Write the expression as the sine, cosine, or tangent of the angle and find the exact value: (so basically, match to one of the formulas on the first page)

a. $\sin 42^\circ \cos 12^\circ - \cos 42^\circ \sin 12^\circ$

$= \sin(42^\circ - 12^\circ) = \sin(30^\circ) = \frac{1}{2}$

b. $\cos 27^\circ \cos 18^\circ - \sin 27^\circ \sin 18^\circ$

$= \cos(27^\circ + 18^\circ) = \cos(45^\circ) = \frac{\sqrt{2}}{2}$

c. $\dfrac{\tan 75^\circ - \tan 15^\circ}{1+\tan 75^\circ \tan 15^\circ}$ $= \tan(75^\circ - 15^\circ) = \tan(60^\circ) = \sqrt{3}$

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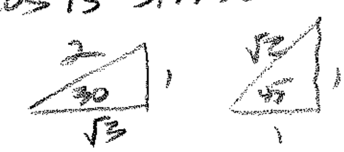
The goal on this next set of problems is to find two angles with REFERENCE angles of 30°, 45° or 60° that can be combined to get the angle you need.

Ex. 1) Find the EXACT value of the following – no decimals!

a. $\sin 75°$ $= \sin(45°+30°) = \sin 45° \cos 30° + \cos 45° \sin 30°$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

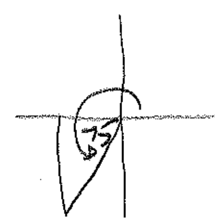


b. $\cos 255°$ $= -\cos 75° = -\cos(45°+30°)$

$= -(\cos 45° \cos 30° - \sin 45° \sin 30°)$

$= -\left(\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right)$

$= \frac{\sqrt{2}-\sqrt{6}}{4}$



c. $\tan 105°$

$= -\tan 75° = -\tan(30°+45°) = -\left(\frac{\tan 30° + \tan 45°}{1-\tan 30° \tan 45°}\right)$

$= -\left(\frac{\frac{\sqrt{3}}{3}+1}{1-\frac{\sqrt{3}}{3}\cdot 1}\right) = -\left(\frac{\frac{\sqrt{3}+3}{3}}{\frac{3-\sqrt{3}}{3}}\right) = \frac{\sqrt{3}+3}{\sqrt{3}-3}\left(\frac{\sqrt{3}+3}{\sqrt{3}+3}\right) = \frac{3+6\sqrt{3}+9}{3-9} = \frac{12+6\sqrt{3}}{-6}$

$= -2-\sqrt{3}$

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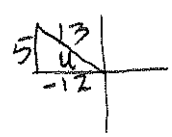
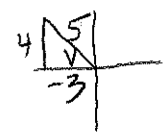




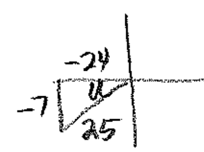
Ex. 3) Find the exact value (no decimals) of the trig function given that

$\sin u=\frac{5}{13}$ and $\cos v=\frac{-3}{5}$ (**both** u and v are in Quadrant II)

$$\sin(v-u)= \sin v\cos u - \cos v \sin u$$

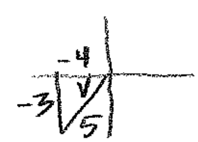
$$= \frac{4}{5}\cdot\frac{-12}{13} - \frac{-3}{5}\cdot\frac{5}{13}$$

$$= \frac{-48}{65} - \frac{-15}{65} = -\frac{33}{65}$$



Ex. 4) Find the exact value of the trig function given that $\sin u=\frac{-7}{25}$ and $\cos v=\frac{-4}{5}$.

If both u and v are in the same quadrant, they must be in Quadrant <u>III</u>



$$\tan(u-v)=\frac{\tan u-\tan v}{1+\tan u\tan v}$$

$$=\frac{\frac{7}{24}-\frac{3}{4}}{1+\frac{7}{24}\cdot\frac{3}{4}} = \frac{\frac{7-18}{24}}{\frac{96+21}{96}}$$

$$= -\frac{11}{\cancel{24}_{1}}\cdot\frac{\cancel{96}^{4}}{117} = -\frac{44}{117}$$

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Ex. 5) Verify the Identity (Yes, it's back!)

$$\cos(x+y)+\cos(x-y)=2\cos x\cos y$$

$$= [\cos x\cos y - \sin x \sin y] + [\cos x \cos y + \sin x \sin y]$$

$$= 2\cos x\cos y \checkmark$$

Ex. 6) Solve the equation (Yes, it's back too!!)

$$\sin 2x\cos x+\cos 2x\sin x=\frac{1}{2}$$

$$= \sin(2x+x) = \frac{1}{2}$$

$$\sin 3x = \frac{1}{2}$$

$$u = 3x$$

$$\sin u = \frac{1}{2}$$

$$u = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$3u = \frac{\pi}{6} \qquad 3u = \frac{5\pi}{6}$$

$$u = \frac{\pi}{18} \qquad u = \frac{5\pi}{18}$$

HW: p. 404-405 #1, 9, 15, 17, 23, 25, 31, 37, 63